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Optimizing The Flexural and Split Tensile Strength Properties of Polystyrene Concrete Using the Osadebe's Model: A Mathematical Approach to Sustainable Environmental and Housing Development.

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Abstract

To its perceived low strength properties, the use of expanded polystyrene beads in concrete applications has been limited. The most of the conclusions in this regard comes from research that used the trial mix method. Given the significant number of trial mixes required to produce a valid result, obtaining a reliable mix ratio using this method is relatively more expensive, difficult, and time consuming. Most studies, on the other hand, have found that demonstrated that utilizing mathematical models to optimize concrete strength qualities to meet various purposes is more dependable and cost-effective. While certain optimization models have successfully predicted various polystyrene concrete strength qualities, only a few research, if any, have used this technique to forecast polystyrene concrete tensile strength. The flexural test is a crucial test that determines whether or not unreinforced concrete beams or slabs can withstand bending failure. The structural rigidity test, on the other hand, is a crucial test that determines if concrete is prone to tensile cracking as a result of the structural load. Both strength tests are crucial in structural design since they establish the tensile strength of the final concrete. The ability of concrete to resist cracking or breaking when under tension is known as tensile strength. Water, sand, coarse aggregates, inflated polystyrene beads, and standard limestone cement were used in this experiment. Except for polystyrene and coarse aggregates, which were stirred together and batched in volume, most materials were batched by weight. The partial replacement level considered was 12% replacement (88% coarse aggregate + 12% polystyrene) using an initial mix ratio of 1:3:6 (cement, sand and coarse aggregate) in accordance to BS EN 1992 for Structural Concrete of 20 N/mm^2. The constituents were manually mixed in the laboratory to prepare 20 different mixes. The results obtained were used for developing the predictor and optimization models respectively. All specimens were cured in accordance with NIS 87 (2004). The laboratory flexural and split tensile strength test results for the 28th day were obtained. Most of the model results agreed with their respective laboratory experiments for both flexural and split tensile strength, particularly for mix1, mix4, mix5, mix6, mix8, mix9 and mix10. The optimized results yielded a flexural and split tensile strength results of 2.00 N/mm^2 and 4.9 N/mm^2 from a water, cement, sand and coarse aggregate mix ratio of 0.449, 1, 2.77 and 5.52 respectively at a 71% water absorption rate. The optimized result exceeds the minimum flexural and split tensile test results as specified by BS EN 12390 – 6 (2009). This has shown that polystyrene lightweight concrete can attain a concrete strength that is suitable for residential purposes and can also be used as partitions in high rising buildings due to their light weight property. If adopted, this model can help in achieving the UN SDG 11, which advocates for sustainable cities and communities, by providing affordable housing.

Keywords: Polystyrene, Lightweight-Concrete, Mathematical-Model, Optimization, Osadebe, Flexural-Strength and Tensile-Strength.

1. Introduction

Plastic waste generation has become more intense in recent times. This has drawn concerns from all quarters with regards to its effect on the environment. Despite the increasing

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concerns, the major source of plastic waste, human population has remained in an upward projectile in terms of growth. This implies that as long as there is continuous human population growth, plastic waste generation is bound to increase. In 2016, the world generated over 242 million tonnes of plastic waste, according to Kaza, Bhada-Tata, and Van Woerden (2018). This loosely translated to over 12 percent of the entire municipal waste. Currently it is estimated to be well above 300 million tonnes every year (UNEP, 2021). The most common of them are single use plastics such as expanded polystyrene (EPS) (commonly used for protective packaging, hot drinks cups etc.), High—density polyethylene (HDPE) Polyethylene terephthalate (PET) (dispensing containers, water, bottles, biscuit trays, etc.), Low-density polyethylene (LDPE) (food packaging film, bags, trays, containers), Polypropylene (PP) and Polystyrene (PS) (cutlery, plates and cups). The majority of these single-use plastics end up as mismanaged plastic garbage in the environment. Over 6.3 billion tonnes of plastic garbage have been produced since the 1950s. According to The Economist (2018), only 9% of this waste generated has been generated, while another 12% incinerated, leaving about 79% in the environment. In 2015 alone, it is reported that between 60 and 99 million metric tons of mismanaged plastic waste were dumped into the environment. This trend has created a growing global environmental concern (Kaza et. al., 2018). While efforts to limit the quantity of plastic garbage generated or develop environmentally friendly alternatives are ongoing, an immediate solution to the current crisis is required. Integrating some of this plastic materials into construction materials during construction seem a more reliable approach, provided they meet the industrial standards for construction. Studies have shown that expanded polystyrene beads (EPS) can integrate with concrete to form what is known as polystyrene lightweight concrete. According (Babu & Babu, 2003), any concrete with a density lesser than or equalling 1800 kg/m^3 can be generally accepted as lightweight concrete. Lightweight concrete produced using expanded polystyrene is generally referred to as lightweight aggregate concrete.

When compared to ordinary concrete, expanded polystyrene (EPS) is a suitable structural material because of its low density, energy absorption, and thermal insulation (Babu & Babu, 2003; Chen & Liu, 2004; Miled et al., 2007). Building construction, highway foundations, floating marine platforms, and military defensive protections are all alternate uses. The mechanical properties of EPS concrete may differ from those of normal concrete because to its ultra-low density and the amazingly smooth and rounded shape of the EPS particles. For this form of concrete, it is necessary to investigate the properties and build a model for making estimates. The mechanical properties of EPS lightweight aggregate concrete have been examined in different of studies (Babu & Babu, 2003; Babu & Babu., 2004; Bouvard et al., 2007; Chen and Liu, 2004, 2007, 2013; Kan and Demirboga, 2009; Sadrmomtazi et al., 2012). The majority of this topic's study is on judging and improving the strength of EPS concrete, such as compressive, flexural, and splitting tensile strength. Meanwhile, some research has provided models for predicting the compressive strength of concretes like standard and self-compacting concrete (Ubi, 2021; Aslani, 2013; Chidiac et al., 2013). However, little work has been put into generating a model for determining the elastic modulus of EPS concrete. Although Le Roy et al., (2005) and Miled et al., (2007, 2011) investigated the effect of EPS bead size on the elastic modulus of EPS lightweight concretes with a mono water/cement (w/c) ratio of 0.26, the properties of EPS lightweight concretes with different w/c ratios (one of the most critical factors affecting material properties) have not been fully reviewed.

The primary reason why there has been a prolonged drought in gainful knowledge concerning the use of EPS in concrete production for modern structural designs has been the issue of low strength. It is well known that the strength of the aggregate material determines the overall strength of the concrete and the strength of EPS beads in itself is near zero. However, new techniques that are capable of attaining total recycling of EPS beads in concrete that specifically aim at modifying them thermally can be explored (Kan, & Demirboğa, 2009). Expanded polystyrene (EPS) beads are cellular lightweight plastics material that consists of well-arranged spherically shaped particles that are composed of about 98% air and 2% polystyrene (Tamut, Prabhu, Venkataramana, & Yaragal, 2014). Its cell structure is closed; hence it is difficult for it to absorb water. It is characteristically well suited for impact resistance as well as thermal and sound insulation. Although polystyrene foam is non-biodegradable, it is extensively used in the packaging business as a packaging medium for fragile objects. They are widely available since they are regularly used to package household items such as electronics. Also, since they are non-biodegradable, their availability has created the disposal challenge. Thus, the use of polystyrene in concrete is also a valuable means of disposing of this industrial waste that is already posing a severe danger to the environment. Usually, this is achieved through trial mixes on a trial-and-error basis. This method is generally difficult, pain staking, time consuming and generally expensive, especially when more concrete components are involved. Hence, the need for polystyrene concrete mix optimization method using mathematical models that can accurately predict the right material proportions and resultant strength. Therefore, the aim of this study is to develop a mathematical model for the prediction and optimization of the flexural and split tensile strength properties of polystyrene lightweight concrete, using the Osadebe's second degree polynomial regression model. Therefore, this study was conducted with two main achievements in view. Firstly, from the environmentalist perspective of solving the challenges of waste disposal, and secondly from the perspective of the construction industry, to provide an alternative to natural aggregates in line with the UN SDG goal 11, which is aimed at developing sustainable cities and communities, by providing affordable housing.

Concrete's tensile skill is the ability to resist cracking or breaking when it is under tension. Despite the fact that concrete is rarely loaded at pure pressure in a structure, determining the tensile strength is vital to determine the amount of the potential damage. Tensile forces exceed the tensile strength, causing breaking and fracture. It is a crucial concrete strength property that has a significant impact on the extent and magnitude of cracking in structures. Due to its low tensile strength and brittle nature, concrete is not normally expected to withstand direct tension. Concrete's tensile strength is substantially lower than its compressive strength (thus the necessity of steel to carry tension stresses). Concrete's tensile strength is believed to be around 10% of its compressive strength. To determine the tensile strength, indirect methods are applied due to the difficulty of the direct method.

2. Derivation of Osadebe's Model.

$$Z_1 + Z_2 + \cdots + Z_q = \textstyle\sum_{i=1}^{q} Zi = 1 \qquad (1)$$

Where q is the number of mixture components and Z_i the proportion of the components in the mixture.

Z_1= Water/Cement Ratio
Z_2 = Binder (Cement)
Z_3 = Fine Aggregates (Sand)
Z_4 = Coarse Aggregates (88% Granite + 12% EPS)

Osadebe assumed that the response Y is continuous and differentiable with respect to its predictors and can be expanded in the neighbourhood of a chosen point Z_o using Taylor's series.

$$Z(0) = (Z_1^{(0)}, Z_2^{(0)}, \dots, Z_q^{(0)})^r \qquad (2)$$

$$Y(z) = \textstyle\sum_{m=0}^{q} F^m\,(Z)^{(0)}\big(Z_i - Z^{(0)}\big) \qquad (3)$$

Expanding to second order

$$Y(Z) = F(Z^{(0)}) + \sum_{i=1}^{q} \frac{\partial f\big(Z^{(0)}\big)}{\partial Z_i}\big(Z_i - Z^{(0)}\big) + \frac{1}{2!}\sum_{i=1}^{q-1}\sum_{i=1}^{q} \frac{\partial^2 f(Z^0)}{\partial Z_i \partial Z_i}(Z_i - Z_i{}^{(0)})(Z_i - Z_i{}^{(0)})$$

$$+ \textstyle\sum_{i-1}^{q} \frac{\partial^2 f(Z^{(0)})}{\partial Z_i}\big(Z_i - (o)\ \big) \qquad (4)$$

For convenience, the point Z^0 can be taken as the origin without loss in generality of the formulation and thus;

$$Z_1{}^{(0)} = Z_1{}^{(0)} + Z_2{}^{(0)} + Z_3{}^{(0)} + \ \dots, Z_q{}^{(0)} = 0 \qquad (5)$$

Let:

$$b_0 = F(0), b_i = \frac{\partial F(0)}{\partial Z_i}, b_{ij} = \frac{\partial^2 F(0)}{2i\partial Z_i \partial j}, b_{ii} = \frac{\partial^2 F(0)}{2i\partial Z_i^2} \qquad (6)$$

Substituting Equation (3.13) into Equation (3.8) gives:

$$Y(Z) = b_0 + \textstyle\sum_{i-1}^{q} b_i Z_i + \sum_{i \le j \le q}^{q} b_{ij} Z_i Z_j + \sum_{i-1}^{q} b_{ii} Z_i^2 \ (7)$$

Multiplying Equation (3.8) by b_0 gives the expression:

$$b_0 = b_0 Z_1 + b_0 Z_2 + \cdots \ldots + b_0 Z_q \qquad (8)$$

Multiplying Equation (3.8) successively by $Z_1, Z_2 \dots Z_q$ and rearranging, gives respectively:

$$Z_1^2 = Z_1 - Z_1 Z_2 - \cdots \dots \dots + Z_1 Z_q$$
$$Z_2^2 = Z_2 - Z_1 Z_2 - \cdots \dots \dots - Z_2 Z_q$$
$$Z_q^2 = Z_1 - Z_1 Z_q - \cdots \dots \dots - Z_{(q-1)} \qquad (9)$$

Substituting Equations (3.13) and (3.15) into Eq. (3.16) and simplifying yields

$$Y(Z) = \textstyle\sum_{i-1}^{q} \beta_i Z_i + \sum_{i \le j \le q}^{q} \beta_{ij} Z_i Z_j \qquad (10)$$

Where

$$\beta_i = b_0 + b_i \dots \dots + b_{ii} \qquad (11)$$

$$\beta_{ij} = b_{ij} - b_{ii} - b_{ij} \qquad (12)$$

If the unknown constant coefficients I and _ij are uniquely determined, Osadebe's regression model equation is defined. The following is the regression equation: The regression equation is: If the number of elements, q, is 4, and the polynomial's degree, m, is 2, the regression equation is:

$$Y = \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_3 + \beta_4 Z_4 + \beta_{12} Z_1 Z_2 + \beta_{13} Z_1 Z_3 + \beta_{14} Z_1 Z_4 + \beta_{23} Z_2 Z_3 + \beta_{24} Z_2 Z_4 + \beta_{34} Z_3 Z_4 \qquad (13)$$

Equation 13 is the mathematical model based on Osadebe's method of second-degree regression.

3. Determination of the Osadebe regression equation coefficients

N is the smallest number of experimental runs or independent answers required to calculate the Osadebe regression coefficients.

Let $y^{(k)}$ be the response at point *k* and the vector corresponding to the set of component proportions (predictors) at point *k* be $Z^{(k)\cdot}$ That is:

$$Z^{(k)} = \{ Z_1{}^{(k),} Z_2{}^{(k)}, \dots\dots, Z_q{}^{(k)} \} \qquad (14)$$

Equation (14) is obtained by substituting the vector of Equation (13) into Equation (14):

$$y^{(k)} = \sum_{i=1}^{q} \beta_i Z_i^{(k)} + \sum_{i \le j \le q}^{q} \beta_{ij} Z_i^{(k)} Z_j^{(k)} \ \ k = 1,2,\dots,N \ (15)$$

When the predictor vectors at each of the N observation points are successively inserted into Equation (15), a set of N linear algebraic equations is constructed, which can be written in matrix form as:

$$\boldsymbol{Z\beta = y} \qquad (16)$$

Where

$\boldsymbol{\beta}$ is a vector whose members are the regression coefficient estimates.The mixture component proportions and functions of the component proportions are the elements of Z, which is a N x N matrix.

y is a vector containing the observations or responses at each of the N observation points.

That is:

$$\boldsymbol{Z}=\begin{bmatrix} Z_1^{(1)} & Z_2^{(1)} & \cdots & Z_1^{(1)}Z_2^{(1)} & \cdots & Z_1^{(1)}Z_q^{(1)} & \cdots & Z_{q-1}^{(1)}Z_q^{(1)} \\ Z_1^{(2)} & Z_2^{(2)} & \cdots & Z_1^{(2)}Z_2^{(2)} & \cdots & Z_1^{(2)}Z_q^{(2)} & \cdots & Z_{q-1}^{(2)}Z_q^{(2)} \\ \vdots & \vdots & & & & & & \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Z_1^{(N-1)} & Z_2^{(N-1)} & \cdots & Z_1^{(N-1)}Z_2^{(N-1)} & \cdots & Z_1^{(N-)}Z_q^{(N-1)} & \cdots & Z_{q-1}^{(N-1)}Z_q^{(N-1)} \\ Z_1^{(N)} & Z_2^{(N)} & & Z_1^{(N)}Z_2^{(N)} & \cdots & Z_1^{(N)}Z_q^{(N)} & & Z_{q-1}^{(N)}Z_q^{(N)} \end{bmatrix}$$

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \vdots \\ \beta_q \\ \beta_{12} \\ \beta_{13} \\ \vdots \\ \beta_{1q} \\ \vdots \\ \\ \beta_{q-1q} \end{bmatrix} \text{ and } \boldsymbol{y} = \begin{bmatrix} y^1 \\ y^2 \\ \vdots \\ \vdots \\ y^q \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ y^N \end{bmatrix}$$

Equation (16) has the following solution: $\boldsymbol{\beta} = \boldsymbol{Z}^{-1}\boldsymbol{y}$ (17)

Table 1: Actual (Si) and (Zi) component fractions for Osadebe's Regression Model.

S/N	Mix ratios				Component's fraction			
	Water	Cement	Sand	Coarse /EPS	Water	Cement	Sand	Coarse /EPS
	S_1	S_2	S_3	S_4	Z_1	Z_2	Z_3	Z_4
1	0.35	1	1	2	0.08	0.229885	0.229885	0.45977
2	0.44	1	1.5	3	0.074	0.16835	0.252525	0.505051
3	0.45	1	2	3	0.07	0.155039	0.310078	0.465116
4	0.5	1	3	6	0.048	0.095238	0.285714	0.571429
5	0.43	1	2	4	0.058	0.13+459	0.269179	0.538358
6	0.48	1	2.5	5	0.053	0.111359	0.278396	0.556793
7	0.51	1	4	6	0.044	0.086881	0.347524	0.521286
8	0.33	1	3	5	0.035	0.107181	0.321543	0.535906
9	0.55	1	2	5	0.064	0.116959	0.233918	0.584795
10	0.6	1	2.5	6	0.059	0.09901	0.247525	0.594059
Control Points								
11	0.55	1	1	2	0.121	0.21978	0.21978	0.43956
12	0.6	1	1.5	3	0.098	0.245902	0.245902	0.491803
13	0.44	1	2	4	0.059	0.134409	0.268817	0.537634
14	0.5	1	2.5	5	0.056	0.277778	0.277778	0.555556
15	0.4	1	3	6	0.038	0.096154	0.288462	0.576923
16	0.43	1	3.5	6.5	0.038	0.306212	0.306212	0.568679
17	0.35	1	4	7	0.028	0.080972	0.323887	0.566802
18	0.51	1	4.5	7.5	0.038	0.333087	0.333087	0.555144
19	0.48	1	4.8	7.6	0.035	0.072046	0.345821	0.54755
20	0.47	1	5	8	0.032	0.345543	0.345543	0.552868

Table 2: Component proportion to determine the coefficients of the Osadebe's regression

Flexural Strength	Split Tensile Strength	Cost Per M3
-1839.1	-1191.2	-1818500.00
558.46	346.91	627840.00
938.23	723.54	554490.00
-502.78	-364.13	-456980.00
1578.6	1000.3	723250.00
1672.4	1143.2	200910.00
9911.6	7004.6	10077000.00
-2729.7	-2043.2	-678520.00
-4650.1	-3276	-5049700.00
698.79	431.69	1116200.00

3. Materials and Methods

For this work, two methodological techniques were used: empirical modeling design and experimental procedures. Water, sand, coarse aggregates, regular limestone cement, and expanded polystyrene beads were among the components employed. The cement was acquired from a local cement wholesaler in Calabar, Cross River State, Nigeria, and was the Lafarge regular limestone cement. For specimen mixing and cures, potable water conforming to EN, BS 1008 (2002) was utilized, as required by NIS 87. (2004). Sand was collected from the seashore beside the Calabar River. S and V quarry limited in Akamkpa, Cross River State, Nigeria, provided granite chippings (coarse aggregates). In Owerri, Imo State, Nigeria, EPS beads were obtained from a local distributor. The materials were batched and mixed together as a single material in volume, excluding coarse aggregates and polystyrene beads, which were batched and mixed together as a single material in volume, bringing the total number of components to four. The mathematical analysis and model development were carried out using Minitab, SPSS, and MATLAB software. In MATLAB, the Predictor and Optimixer programs were created to anticipate model results and determine optimal mixes using equations 13, 16, and 17. The various ingredients were manually combined in the laboratory, and the findings were based on the 28th day test for model optimization. The partial replacement level used was 12 percent (88 percent coarse aggregate + 12 percent polystyrene) with a mix ratio of 1:3:6 (cement, sand, and coarse aggregate) in line with EN BS 882 (1992) for 20 N/mm2 structural concrete. EN, BS 1008 (2002), and NIS 87 were used to cure all specimens (2004). The experiment was carried out at the Cross River University of Technology, Calabar, Nigeria, at the Strength of Material Lab, Workshop 5. In order to assess the flexural strength, sixty 500 mm X 100 mm sample beams were moulded. For the flexural strength test, three layers of newly mixed concrete were poured into each mould. Each layer of concrete was hand compacted 150 times with a 25 mm steel rod. In order to assess the split tensile strength, sixty 100 mm by 200 mm cylindrical concrete specimens were also moulded. Freshly mixed concrete was placed into each mould in two layers of around 100 mm thickness for the split tensile strength. Each layer was manually compacted by tamping the rod 35 times on each layer with 25 mm steel rods. Following that, in accordance with BS EN 12390 – 6, the hardened sample beams were placed on the testing machine (2009).

4. Discussion of Findings and Results

4.1. Empirical Findings for Flexural Strength

Equation 17 is used to fit the data in Table 3 to obtain the model predictor (equation 18). The model equation was developed, and the polynomial coefficients were obtained, and the F statistic was used to check for goodness.

$\beta_1 = -1839.1, \beta_2 = 558.46, \beta_3 = 938.23, \beta_4 = -502.78, \beta_{12} = 1578.6, \beta_{13} = 1672.4, \beta_{14} = 9911.6, \beta_{23} = -2729.7, \beta_{24} = -4650.1, \beta_{34} = 698.79$ -

As a result, Osadebe's second degree flexural strength model's regression equation is:

$$Y = -1839.1Z_1 + 558.46Z_2 + 938.23Z_3 - 502.78Z_4 + 1578.6Z_1Z_2 + 1672.4Z_1Z_3 + 9911.6Z_1Z_4 - 2729.7Z_2Z_3 - 4650.1Z_2Z_4 + 698.79Z_3Z_4 \tag{18}$$

Verify for a lack of fit.

Table 4 reveals that the lack-of-fit is insignificant, with a p-value of 0.00, which is less than 0.05. As a result, Equation 17 is appropriate for forecasting the flexural strength of expanded polystyrene concrete on the 28th day. The model's adequacy is supported by the additional statistics in Table 5.

Table 3: Flexural Strength Regression Coefficients Estimated.

Flexural Strength						
Model		Unstandardized Coefficients		Standardized Coefficients	T	Sig.
		B	Std. Error	Beta		
1	Z_1	-1839.1	43.488	-.053	-.093	.928
	Z_2	558.46	14.039	2.995	3.383	.006
	Z_3	938.23	5.663	2.992	4.507	.001
	Z_4	-502.78	76.123	.081	.406	.693
	$Z_1 * Z_2$	1578.6	134.699	.830	1.761	.106
	$Z_1 * Z_3$	1672.4	99.991	-1.123	-1.656	.126
	$Z_1 * Z_4$	9911.6	30.774	-.653	-1.719	.114
	$Z_2 * Z_3$	-2729.7	21.755	.596	1.227	.246
	$Z_2 * Z_4$	-4650.1	23.504	-4.756	-5.932	.000
	$Z_3 * Z_4$	698.79	43.488	-.053	-.093	.928

Table 4: Flexural Strength ANOVA.

Model		Sum of Squares	Df	Mean Square	F	Sig.
1	Regression	420.247	9	46.694	720.721	.000c
	Residual	.713	11	.065		
	Total	420.960d	20			

Table 5: Residuals Statistics for Flexural Strength.

	Minimum	Maximum	Mean	Std. Deviation	N
Predicted Value	2.569116	6.455096	4.478639	1.0021787	20
Residual	-.2851150	.3814826	.0001820	.1936720	20
Std. Predicted Value	-1.905	1.972	.000	1.000	20
Std. Residual	-1.120	1.499	.001	.761	20

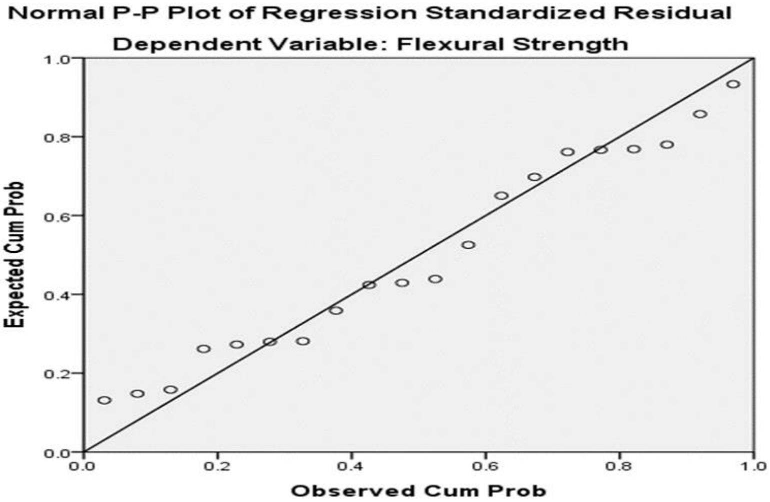
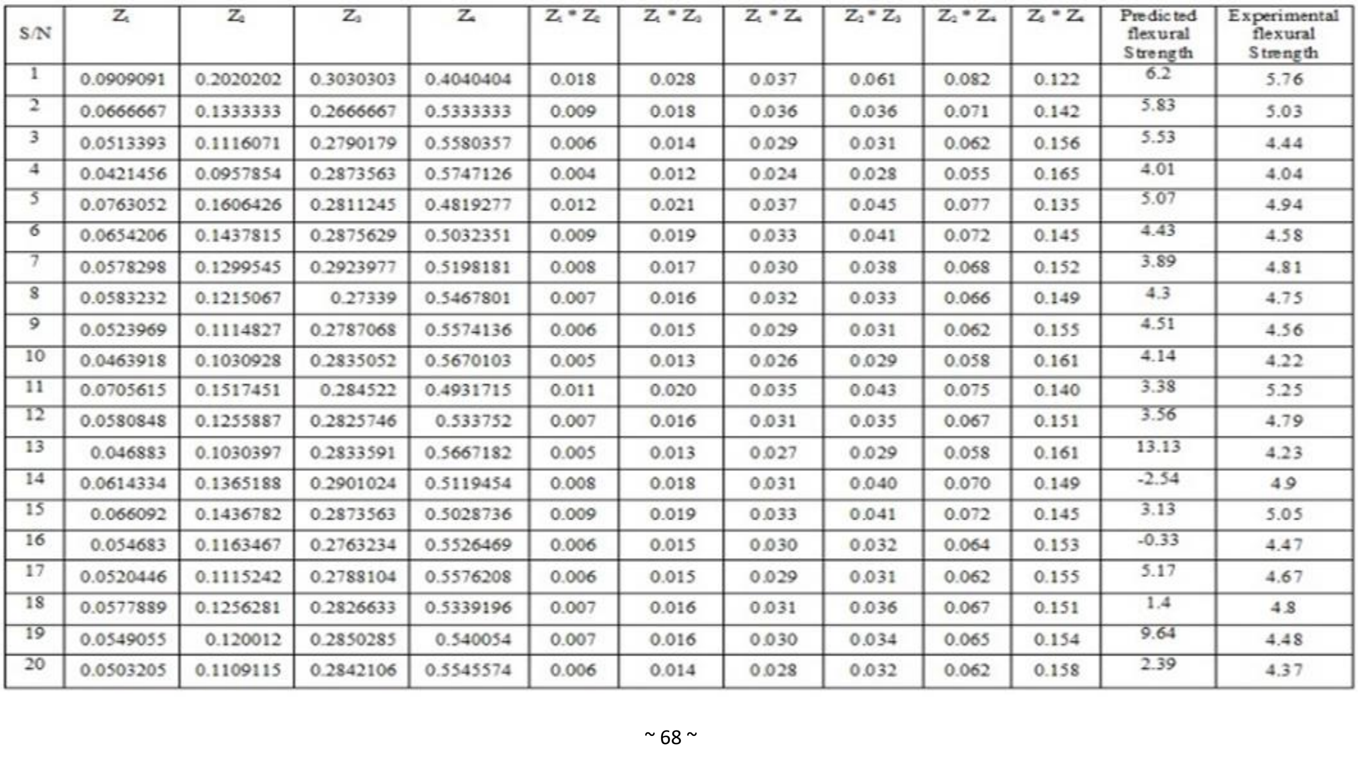


Fig.1: Normal probability plot for flexural strength residuals.

4.1.1. Flexural strength model predictions.

The model results in Table 6 were predicted using the flexural strength model equation (equation 18). The model results agree with the experimental data to a good amount, especially for mix 1, mix 4, mix 5, mix 9, and mix 10. Mix 1 has the highest strength of the five, with nearly 6.2 N/mm2. When analysing mixes with negative strength values in the optimixer, it is believed that such mixes will provide poorer strength findings. Mix 14 and mix 16 yielded the following results. It's also worth noting that the anticipated results can't be used in the real world until they've been converted back to actual ratios. The results are also depicted in figure 2 as a graphical example.

Table 6: Model Predicted Result for Flexural Strength



S/N	Z_1	Z_2	Z_3	Z_4	$Z_1 * Z_2$	$Z_1 * Z_3$	$Z_1 * Z_4$	$Z_2 * Z_3$	$Z_2 * Z_4$	$Z_3 * Z_4$	Predicted flexural Strength	Experimental flexural Strength
1	0.0909091	0.2020202	0.3030303	0.4040404	0.018	0.028	0.037	0.061	0.082	0.122	6.2	5.76
2	0.0666667	0.1333333	0.2666667	0.5333333	0.009	0.018	0.036	0.036	0.071	0.142	5.83	5.03
3	0.0513393	0.1116071	0.2790179	0.5580357	0.006	0.014	0.029	0.031	0.062	0.156	5.53	4.44
4	0.0421456	0.0957854	0.2873563	0.5747126	0.004	0.012	0.024	0.028	0.055	0.165	4.01	4.04
5	0.0763052	0.1606426	0.2811245	0.4819277	0.012	0.021	0.037	0.045	0.077	0.135	5.07	4.94
6	0.0654206	0.1437815	0.2875629	0.5032351	0.009	0.019	0.033	0.041	0.072	0.145	4.43	4.58
7	0.0578298	0.1299545	0.2923977	0.5198181	0.008	0.017	0.030	0.038	0.068	0.152	3.89	4.81
8	0.0583232	0.1215067	0.27339	0.5467801	0.007	0.016	0.032	0.033	0.066	0.149	4.3	4.75
9	0.0523969	0.1114827	0.2787068	0.5574136	0.006	0.015	0.029	0.031	0.062	0.155	4.51	4.56
10	0.0463918	0.1030928	0.2835052	0.5670103	0.005	0.013	0.026	0.029	0.058	0.161	4.14	4.22
11	0.0705615	0.1517451	0.284522	0.4931715	0.011	0.020	0.035	0.043	0.075	0.140	3.38	5.25
12	0.0580848	0.1255887	0.2825746	0.533752	0.007	0.016	0.031	0.035	0.067	0.151	3.56	4.79
13	0.046883	0.1030397	0.2833591	0.5667182	0.005	0.013	0.027	0.029	0.058	0.161	13.13	4.23
14	0.0614334	0.1365188	0.2901024	0.5119454	0.008	0.018	0.031	0.040	0.070	0.149	-2.54	4.9
15	0.066092	0.1436782	0.2873563	0.5028736	0.009	0.019	0.033	0.041	0.072	0.145	3.13	5.05
16	0.054683	0.1163467	0.2763234	0.5526469	0.006	0.015	0.030	0.032	0.064	0.153	-0.33	4.47
17	0.0520446	0.1115242	0.2788104	0.5576208	0.006	0.015	0.029	0.031	0.062	0.155	5.17	4.67
18	0.0577889	0.1256281	0.2826633	0.5339196	0.007	0.016	0.031	0.036	0.067	0.151	1.4	4.8
19	0.0549055	0.120012	0.2850285	0.540054	0.007	0.016	0.030	0.034	0.065	0.154	9.64	4.48
20	0.0503205	0.1109115	0.2842106	0.5545574	0.006	0.014	0.028	0.032	0.062	0.158	2.39	4.37

Fig.2: Comparison of the model predicted flexural strength results against the actual laboratory results.

4.2. Split Tensile Strength Empirical Findings

Equation 19 is obtained by plugging the data from Table 7 into equation 17. The model equation was developed, and the polynomial coefficients were obtained, and the F statistic was used to check for goodness.

$\beta_1 = -1191.2, \beta_2 = 346.91, \beta_3 = 723.54, \beta_4 = -364.13, \beta_{12} = 1000.3, \beta_{13} = 1143.2, \beta_{14} = 7004.6, \beta_{23} = -2043.2, \beta_{24} = -3276, \beta_{34} = 431.69$ -

As a result, Osadebe's second degree model for split tensile strength's regression equation is:

$$Y = -1191.2Z_1 + 346.91Z_2 + 723.54Z_3 - 364.13Z_4 + 1000.3Z_1Z_2 + 1143.2Z_1Z_3 + 7004.6Z_1Z_4 - 2043.2Z_2Z_3 - 3276Z_2Z_4 + 431.69Z_3Z_4 \text{ -----} \quad (19)$$

Check for a lack of fit

The lack-of-fit is insignificant, shown in Table 8, with a p-value of 0.00, which is less than 0.05. As a result, equation 19 is used to calculate the split tensile strength of expanded polystyrene lightweight concrete on day 28. The additional statistics in Table 9 support the model's adequacy.

Table 7: Tensile Strength Regression Coefficients Estimated.

Tensile Strength						
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	Z_1	-1191.2	27.532	-.741	-.778	.453
	Z_2	346.91	8.888	4.480	3.068	.011
	Z_3	723.54	3.585	3.181	2.906	.014
	Z_4	-364.13	48.193	.186	.568	.582
	$Z_1 * Z_2$	1000.3	85.277	.415	.534	.604
	$Z_1 * Z_3$	1143.2	63.304	-.068	-.061	.953
	$Z_1 * Z_4$	7004.6	19.483	.009	.014	.989
	$Z_2 * Z_3$	-2043.2	13.773	-.099	-.123	.904
	$Z_2 * Z_4$	-3276	14.880	-6.468	-4.893	.000
	$Z_3 * Z_4$	431.69	27.532	-.741	-.778	.453

Table 8: ANOVA for Tensile Strength.

Model		Sum of Squares	Df	Mean Square	F	Sig.
1	Regression	61.763	9	6.863	264.274	.000c
	Residual	.286	11	.026		
	Total	62.049d	20			

Table 9: Residuals Statistics for Tensile Strength.

	Minimum	Maximum	Mean	Std. Deviation	N
Predicted Value	1.036031	2.685279	1.688559	.4994000	20
Residual	-.1765737	.2865164	.0001711	.1226128	20
Std. Predicted Value	-1.307	1.996	.000	1.000	20
Std. Residual	-1.096	1.778	.001	.761	20

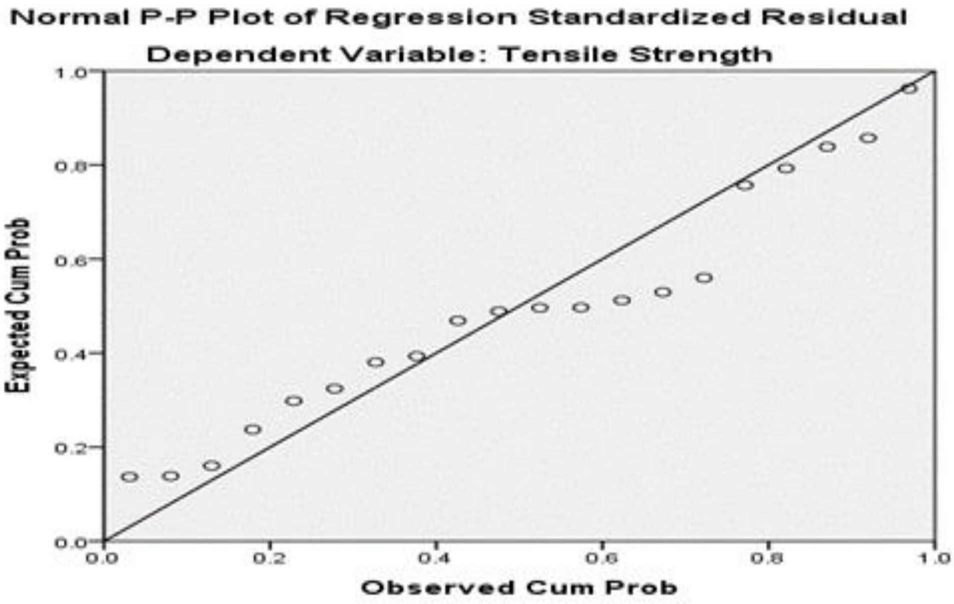


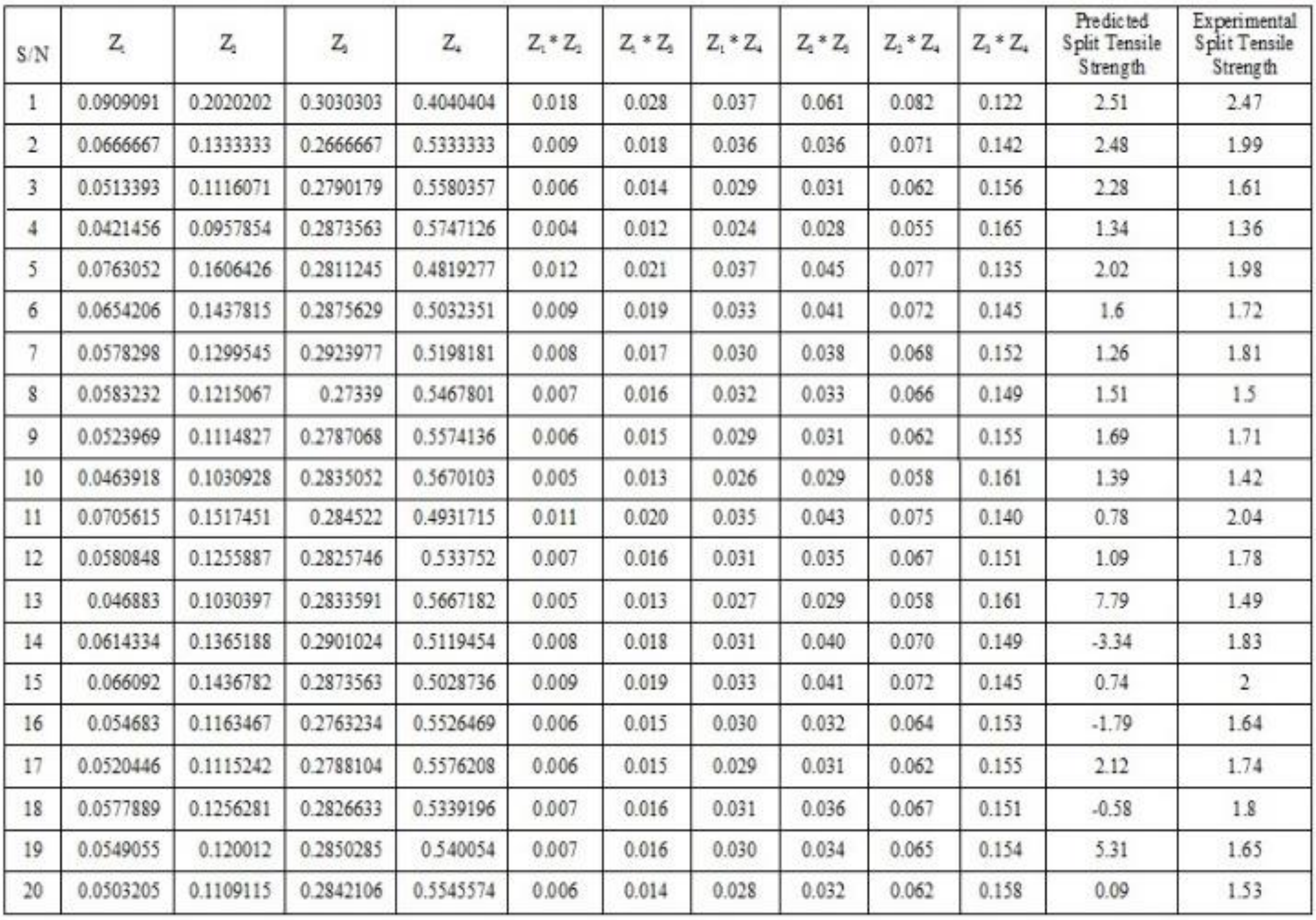
Fig. 3: Normal probability plot for Tensile Strength.

4.2.1. Split tensile strength model predictions.

The model results in Table 10 were predicted using the tensile strength model equation (equation 19). Similarly, to the flexural strength model results, the tensile strength model results agree with the experimental results to a considerable extent, notably for mix 1, mix 4, mix 5, mix 6, mix 8, mix 9 and mix 10. Mix 1 had the highest strength of the seven mixtures, with nearly 2.51 N/mm2. When analysing mixes with negative strength values in the optimixer, it is believed that such mixes will provide poorer strength findings. Mix14, mix16, and mix18 produced these results.

It's also worth noting that the anticipated results can't be used in the real world until they've been converted back to actual ratios. The results are also depicted in figure 4 as a graphical example.

Table 10: Model Predicted Result for Split Tensile Strength.



S/N	Z_1	Z_2	Z_3	Z_4	$Z_1 * Z_2$	$Z_1 * Z_3$	$Z_1 * Z_4$	$Z_2 * Z_3$	$Z_2 * Z_4$	$Z_3 * Z_4$	Predicted Split Tensile Strength	Experimental Split Tensile Strength
1	0.0909091	0.2020202	0.3030303	0.4040404	0.018	0.028	0.037	0.061	0.082	0.122	2.51	2.47
2	0.0666667	0.1333333	0.2666667	0.5333333	0.009	0.018	0.036	0.036	0.071	0.142	2.48	1.99
3	0.0513393	0.1116071	0.2790179	0.5580357	0.006	0.014	0.029	0.031	0.062	0.156	2.28	1.61
4	0.0421456	0.0957854	0.2873563	0.5747126	0.004	0.012	0.024	0.028	0.055	0.165	1.34	1.36
5	0.0763052	0.1606426	0.2811245	0.4819277	0.012	0.021	0.037	0.045	0.077	0.135	2.02	1.98
6	0.0654206	0.1437815	0.2875629	0.5032351	0.009	0.019	0.033	0.041	0.072	0.145	1.6	1.72
7	0.0578298	0.1299545	0.2923977	0.5198181	0.008	0.017	0.030	0.038	0.068	0.152	1.26	1.81
8	0.0583232	0.1215067	0.27339	0.5467801	0.007	0.016	0.032	0.033	0.066	0.149	1.51	1.5
9	0.0523969	0.1114827	0.2787068	0.5574136	0.006	0.015	0.029	0.031	0.062	0.155	1.69	1.71
10	0.0463918	0.1030928	0.2835052	0.5670103	0.005	0.013	0.026	0.029	0.058	0.161	1.39	1.42
11	0.0705615	0.1517451	0.284522	0.4931715	0.011	0.020	0.035	0.043	0.075	0.140	0.78	2.04
12	0.0580848	0.1255887	0.2825746	0.533752	0.007	0.016	0.031	0.035	0.067	0.151	1.09	1.78
13	0.046883	0.1030397	0.2833591	0.5667182	0.005	0.013	0.027	0.029	0.058	0.161	7.79	1.49
14	0.0614334	0.1365188	0.2901024	0.5119454	0.008	0.018	0.031	0.040	0.070	0.149	-3.34	1.83
15	0.066092	0.1436782	0.2873563	0.5028736	0.009	0.019	0.033	0.041	0.072	0.145	0.74	2
16	0.054683	0.1163467	0.2763234	0.5526469	0.006	0.015	0.030	0.032	0.064	0.153	-1.79	1.64
17	0.0520446	0.1115242	0.2788104	0.5576208	0.006	0.015	0.029	0.031	0.062	0.155	2.12	1.74
18	0.0577889	0.1256281	0.2826633	0.5339196	0.007	0.016	0.031	0.036	0.067	0.151	-0.58	1.8
19	0.0549055	0.120012	0.2850285	0.540054	0.007	0.016	0.030	0.034	0.065	0.154	5.31	1.65
20	0.0503205	0.1109115	0.2842106	0.5545574	0.006	0.014	0.028	0.032	0.062	0.158	0.09	1.53

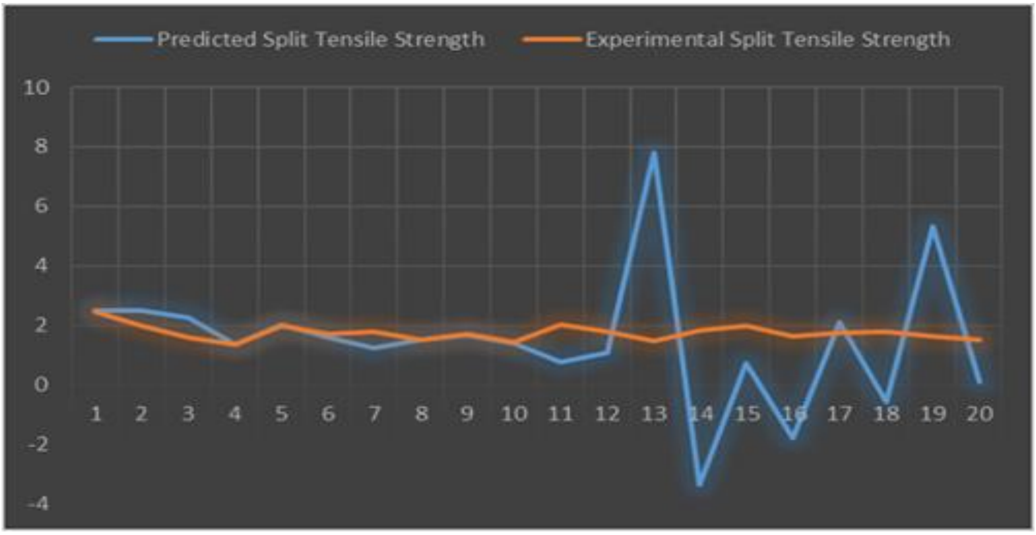
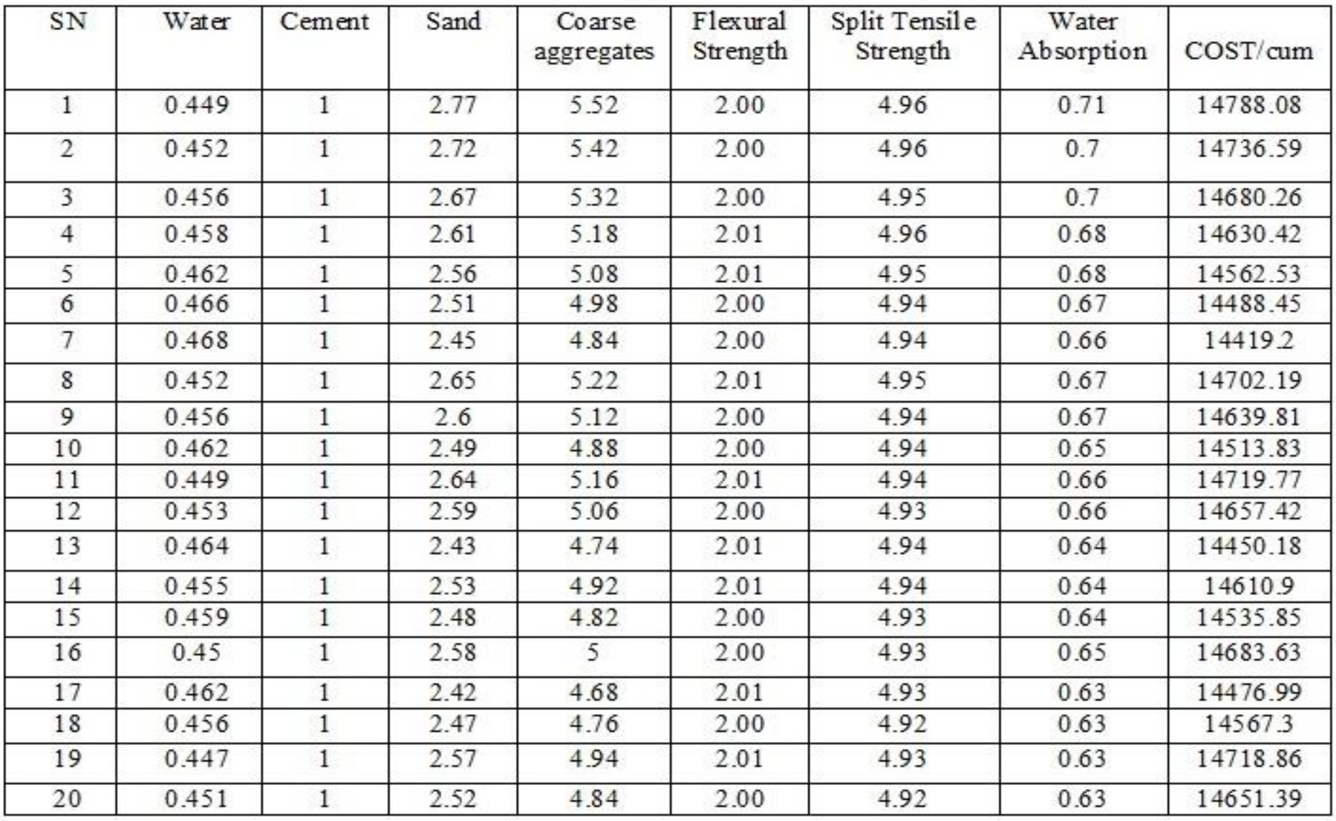


Fig. 4: Comparison of the model predicted tensile strength results against the actual laboratory results.

5. The end result of the optimization.

The optimum mix ratios created by the optimixer based on the projected results are shown in Table 11. From a water, cement, sand, and coarse aggregate (at 88 percent coarse aggregate + 12 percent polystyrene) of 0.449, 1, 2.77, and 5.52, respectively, mix1 will generate a flexural strength of 2N/mm^2 and a split tensile strength of 4.96N/mm^2. Mix1 stands out since it has a higher water absorption rate of 71 percent. The rest of the results are acceptable because none of them achieved a flexural or split tensile strength less than 10% of the 20N/mm^2, as required by BS EN 1992 for Structural Concrete. EN, BS. 206 (2013) and ASTM, C. 39 are both applicable to the optimum outcomes (2001). This means that with a 12 percent partial substitution of coarse aggregates with expanded polystyrene beads, all optimum mixes can generate adequate flexural and split tensile strengths suited for residential and commercial constructions.

Table 11 shows the results of the optimized flexural and split tensile tests.



SN	Water	Cement	Sand	Coarse aggregates	Flexural Strength	Split Tensile Strength	Water Absorption	COST/cum
1	0.449	1	2.77	5.52	2.00	4.96	0.71	14788.08
2	0.452	1	2.72	5.42	2.00	4.96	0.7	14736.59
3	0.456	1	2.67	5.32	2.00	4.95	0.7	14680.26
4	0.458	1	2.61	5.18	2.01	4.96	0.68	14630.42
5	0.462	1	2.56	5.08	2.01	4.95	0.68	14562.53
6	0.466	1	2.51	4.98	2.00	4.94	0.67	14488.45
7	0.468	1	2.45	4.84	2.00	4.94	0.66	14419.2
8	0.452	1	2.65	5.22	2.01	4.95	0.67	14702.19
9	0.456	1	2.6	5.12	2.00	4.94	0.67	14639.81
10	0.462	1	2.49	4.88	2.00	4.94	0.65	14513.83
11	0.449	1	2.64	5.16	2.01	4.94	0.66	14719.77
12	0.453	1	2.59	5.06	2.00	4.93	0.66	14657.42
13	0.464	1	2.43	4.74	2.01	4.94	0.64	14450.18
14	0.455	1	2.53	4.92	2.01	4.94	0.64	14610.9
15	0.459	1	2.48	4.82	2.00	4.93	0.64	14535.85
16	0.45	1	2.58	5	2.00	4.93	0.65	14683.63
17	0.462	1	2.42	4.68	2.01	4.93	0.63	14476.99
18	0.456	1	2.47	4.76	2.00	4.92	0.63	14567.3
19	0.447	1	2.57	4.94	2.01	4.93	0.63	14718.86
20	0.451	1	2.52	4.84	2.00	4.92	0.63	14651.39

6. Conclusion and Recommendation

The main issue with polystyrene lightweight concrete's universal acceptability for construction had been concerns about its poor strength. This was the case because the sole technique of conducting such tests was through trial and error. This mix approach has not shown to be effective due to the difficulty in determining the best mix proportions, especially when multiple components are involved, such as when aggregate is partially replaced with Expanded Polystyrene beads. As a result, as demonstrated in this study, the employment of mathematical models has proven to be more efficient and accurate. Without any more trial mixes, the mix proportions for the appropriate flexural strength for lightweight concrete performance can be simply recreated using this model. The optimum flexural and split tensile strength results achieved at 2.00N/mm2 and 4.9N/mm2, respectively, are in agreement with the minimum tensile strength results defined by BS EN 12390 – 6. (2009). This approach, if implemented, will invariably reduce the time, material, and financial costs of building with polystyrene lightweight concrete, while also helping to reduce the amount of polystyrene-related plastic waste in the environment. It will also contribute to housing affordability and availability, in line with SDG 11 of the United Nations, which advocates for sustainable cities and communities, by providing affordable housing. However, more research in this area is required in order to better confirm the conclusions of this study and maybe produce better results in terms of higher strength attributes.

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